Safe highways platooning with minimized inter-vehicle distances of the time headway policy

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Abstract—Optimizing the inter-distances between vehicles is very important to reduce traffic congestion on highways.

Variable spacing and constant spacing are the two policies for the longitudinal control of platoon. Variable spacing doesn’t require a lot of data (position, speed...) from other vehicles, and string stability using only on-board information is obtained. However, inter-vehicle distances are very large, and hence traffic density is low. Constant spacing can offer string stability with high traffic density, but it requires at least data from the leader.

In [1], we have proposed a modification of the constant time headway control law. This modification leads to inter-vehicle distances that are close to those obtained with constant spacing policies, while requiring only low rate information from the leader.

In this paper, the work done in [1] is extended by taking into account the model of the motor. This enables to reduce the distance between the vehicles to 1 meter, and it has been proved that the platoon is stable and safe in normal working mode. Simulation results are done using TORCS simulator environment.

I. INTRODUCTION

The problems of traffic congestion, pollution, and people safety are becoming more and more important due to the increase in the number of cars.

Proposed solutions to these problems on highways differ from those in urban areas. On highways, road curvature is smaller and there are less obstacles. Under normal conditions, cars move faster than in urban areas.

Some proposed ideas require changes to the infrastructure (automatic speed limits, roads monitoring, reversible lanes...) Other ideas rely on automated vehicles to increase traffic density and to avoid the oscillation of the platoon. Driving in platoon has many advantages: it increases traffic density and to avoid the oscillation of the platoon. Driving in platoon, and have the same sign to avoid collisions.

From the modeling and control point of view, it is possible to decouple the longitudinal and lateral behaviors, when road curvature is assumed to be low, or by using techniques like chained systems theory [15]. Lateral control can be performed using different modalities like 3D laser (as used by the famous Google car), magnetic markers (PATH project), vision sensors [9]... So in a highway environment, it is common to concentrate on longitudinal behavior, including modelling and control.

Platoon models can be found in [13], ranging from systems which do not include communication between the vehicles to systems which use full communications between them. Other authors have build physics-inspired models of the platoon: [2] considers the platoon as a multi agent system, in which the agents (vehicles) interact according to physical phenomena or mimick animal interaction behaviors, [17] represents the interactions as virtual spring-dumper systems, while [5] uses Newton forces.

In platooning applications, the desired behavior of a vehicle is generally defined by a desired distance to the previous vehicle in the platoon. Stability of the platoon control is very important. It uses the concept of String Stability, which requires that distance errors do not amplify as they propagate along the platoon, and have the same sign to avoid collisions. The definition is given in the time domain in [13] and in the frequency domain in [8].

Local control uses data from adjacent vehicles only, while global control depends on data from at least the leader. In local control, the car is totally autonomous: it does not require sophisticated sensors, and can be used in all environments, but trajectory tracking and inter-vehicle distances keeping are not very accurate. On the other hand, global control is more accurate, but it requires more sophisticated sensors, and finally it requires very reliable communication systems.

Two policies are used to control the spacing between vehicles: constant spacing and variable spacing. Variable spacing usually doesn’t require a lot of data from other vehicles. In addition, it can ensure string stability using on-board information only [4], but inter-vehicle distances vary with velocity and can be very large, hence traffic density is low. Constant spacing can achieve both string stability and high traffic density, at the cost of inter-vehicle communications.

Constant Time Headway (CTH) is the simplest and most common variable spacing policy [14], [17]. Variable time headway can vary linearly with the velocity, with relative velocity [18], or even with vehicle dynamics and road conditions [3].

In this paper, we will concentrate on the longitudinal control of platoons on highways. We will propose a modification to the time headway policy, develop the corresponding dynamic control law, study the stability of the platoon and demonstrate the effectiveness and safety of the novel approach for small inter-vehicle distances. The new

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control law is a mixture of local and global decentralized control. This work is a preliminary work to be generalized for platoons working in urban areas. Safety issues due to abnormal working conditions will not be discussed in this paper.

The paper is organized as follows. Section 2 describes the vehicle and platoon models. The control and string stability are presented in section 3. Section 4 explains the simulation results. Finally, section 5 discusses the most important advantages of the proposed approach, and compares it with other existing approaches.

II. MODELING AND CONTROL

In the case of platooning on highways, where the road curvature is small, it is known that longitudinal and lateral controls can be considered as decoupled. In this paper, we also make this safe assumption, which allows us to consider only longitudinal control.

A. Longitudinal Dynamic Model of the Vehicle

According to Newton’s law, we can write the dynamic equation [12] of the vehicle in the platoon shown in figure (1) as:

\[ m \ddot{x} = F + F_g + F_{aero} + F_{drag} \]

\[ m \ddot{x} = F - m g \sin(\theta) - \rho \frac{A C_d}{2} \dot{x}^2 \text{sgn}(\dot{x}) - d_m \] (1)

Since the vehicles are assumed to travel in the same direction at all times then we have \( sgn(\dot{x}) = 1 \).

The engine of the vehicle is modeled as a first degree system, and is given by the following equation

\[ \dot{F} = -\tau F + u \] (2)

So the model of the vehicle can be represented in figure (2):

where:
- \( x \): Position of the vehicle along X axis.
- \( F \): Force produced by the vehicle engine.
- \( \tau \): The vehicle engine time constant.
- \( u \): The control input to the vehicle engine.
- \( F_g, F_{aero}, F_{drag} \): Gravitational, aero-dynamical and mechanical drag force respectively.
- \( g \): Acceleration of gravity.

\[ u = m w + \tau F + m g \cos(\theta) \dot{\theta} - \rho A C_d \dot{x} \dot{x} + u \] (3)

We can use exact linearization to linearize the previous system. We obtain a linear model of the longitudinal dynamics of the car by taking:

\[ \dot{w} = m w + \tau F + m g \cos(\theta) \dot{\theta} - \rho A C_d \dot{x} \dot{x} + u \] (4)

Then, we get:

\[ x^{(3)} = w \] (5)

where \( w \) is the new control input for the linearized system shown in figure (3).

B. Platoon definitions

Figure (4) shows a platoon which consists of \( N \) vehicles required to move at the same speed \( v_d \) with a desired inter-distance \( L \) between two successive vehicles. The leader of the platoon can be driven by a human or autonomously. The followers are controlled to maintain a desired inter-distance.

We define the spacing error of the \( i \)-th vehicle assuming a point mass model for all vehicles:
where:
- \( \Delta X_i = x_i - x_{i-1} \) real spacing between car number \( i \) and its predecessor, car number \( i-1 \).
- \( x_i \): position of \( i \)-th vehicle.
- \( L \): desired inter-vehicle distance.

The kinematic evolution of the spacing error is given by:
\[
\dot{e}_i = \dot{x}_i - \dot{x}_{i-1} = v_i - v_{i-1}
\]
where \( v_i \) represents the velocity of the \( i \)-th vehicle.

## III. PLATOON CONTROL AND STABILITY

### A. Control Objectives

The main objectives of the control law are to:

1) Keep the inter-vehicle distance equal to \( L \), and make all vehicles move at the same speed so \( \dot{e}_i = 0 \).
2) Assure the string stability of the platoon (the spacing error does not increase as it propagates through the platoon).
3) Increase the traffic density.
4) Keep the system stable in case of total loss of communication.

### B. Control Law

In constant spacing control, the control law will make \( e_i \to 0 \) so the inter-vehicle distance will become equal to \( L \). But this requires, at least, information from the leader to assure the string stability of the platoon and robustness.

In time headway policy, a new term is added to the previous error, which will eliminate the need for communication with the leader and increase the string stability. A new spacing error is defined as:
\[
\delta_i = e_i - h (v_i - \bar{v}) = \Delta X_i - L - h (v_i - \bar{v})
\]
where \( \bar{v} \) is a velocity value shared between all the vehicles at the same sampling time.

The new control law is defined by:
\[
u_i = -k_a \ddot{x}_i + k_v \dot{e}_i + k_p \delta_i
\]
which is represented in figure (5) for the \( i \)-th vehicle.

To verify the effectiveness of the new law, the string stability of the platoon under this control law must be analyzed.

### C. String Stability Analysis

The general string stability definition is given in [13]. In essence, it means that all the states are bounded if the initial states (position and velocity errors) are bounded and summable.

A sufficient condition for string stability is given in [8]:
\[
\|e_i\|_{\infty} \leq \|e_{i-1}\|_{\infty}
\]
which means that the spacing error must not increase as it propagates through the platoon. To verify this condition, the spacing error propagation transfer function is defined by:
\[
G_i(s) = \frac{e_i(s)}{e_{i-1}(s)}
\]
A sufficient condition for string stability is given by:
\[
\|G_i(s)\|_{\infty} \leq 1 \quad \text{and} \quad g_i(t) > 0 \quad i = 1, 2, \ldots, N
\]
where \( g_i(t) \) is the error propagation impulse response of the \( i \)-th vehicle.

So, to verify the string stability of a platoon using the novel spacing error, the spacing error propagation transfer function \( G(s) \) must be calculated:
\[ G_1(s) = \frac{k_v s + k_p}{s^3 + k_a s^2 + (k_v + h k_p) s + k_p} \]  

So

\[ ||G_1(\omega)|| = \sqrt{\frac{k_p^2 + k_v^2 \omega^2}{(k_p - k_a \omega^2)^2 + ((k_v + h k_p) \omega - \omega^3)^2}} \]  

To ensure the stability we must verify the condition 7 so we get:

\[ \omega^6 + (k_a^2 - 2(k_a + k_p h)) \omega^4 + (k_v^2 h^2 + 2 k_p (k_v h - k_a)) \omega^2 \geq 0 \]  

(10)

To simplify we choose \( k_v = k_a / h \), which makes the coefficient of \( \omega^2 \) always positive, then the stability conditions become:

\[
\begin{cases}
  h k_a \geq 2 \\
  h k_a^2 - 2 k_a - 4 k_p h^2 \leq 0
\end{cases}
\]  

or \[
\begin{cases}
  h k_a \leq 2 \\
  h k_a^2 - 2 k_a - 4 k_p h^2 \geq 0
\end{cases}
\]

(11)

D. Maximum error amplitudes

In a stable platoon, the maximum error between vehicles is the error between the leader and the first vehicle. If we choose \( V_s = v_{leader} \) then the transfer function of the first error in the platoon is given by:

\[ G_1(s) = \frac{e_1(s)}{w_{leader}(s)} = \frac{1}{s^3 + k_a s^2 + (k_v + h k_p) s + k_p} \]  

(12)

The magnitude of this function is given by:

\[ ||G_1(\omega)|| = \frac{1}{\sqrt{\frac{k_p - k_a \omega^2}{(k_p - k_a \omega^2)^2 + ((k_v + h k_p) \omega - \omega^3)^2}}} \]  

(13)

If the platoon is stable and by choosing \( k_p > 1 \), we get:

\[ ||G_1|| < ||G_i|| \leq 1 \]  

(14)

then

\[ ||e_1|| < ||w_{leader}|| \]  

(15)

So the maximum error in the platoon is bounded by the maximum control value of the leader (the jerk of the leader).

For passenger comfort [16] the maximum value of the jerk should not be bigger than 0.5 – 0.6 m/s^3, so it is clear that we can get a maximum error between vehicle much smaller than 1. So we have proved that we can get a stable platoon system with inter-vehicle distance equal to 1 without collision between vehicles.

IV. SIMULATIONS

The control law has been checked using TORCS. The Open Racing Car Simulator, a software which give us realistic results (as it takes many phenomena into account) and allows visual output when applying the novel spacing error.

TORCS is one of the most popular car racing simulators [7]. It is written in C++ and is available under GPL license from its web page. TORCS presents several advantages for academic purposes, namely:

1) It lies between advanced simulators, like recent commercial car racing games, and a fully customizable environment, like the ones typically used by computational intelligence researchers for benchmark purposes.

2) It features a sophisticated physics engine (aerodynamics, fuel consumption, traction...) as well as a 3D graphics engine for the visualization of the races.

3) It was not conceived as a free alternative to commercial racing games, but it was specifically designed to make it as easy as possible to develop your own controller.

All the simulations were done on nearly straight roads (small curvatures). The desired speed of the leader of the platoon is changed three times (see figure 6), to check the transit response and the stability of the platoon.

At the same time, a comparison between our control law and the classical CTH control law will be performed using the same parameters.

We consider 10 identical vehicles and we choose the following parameters values: \( h = 3, k_v = 1/3, k_p = 5, Ka = 1 \). The desired inter-vehicle distance (bumper-to-bumper distance, so we omit all the cars lengths from all following figures) is fixed to \( L = 1 \) m.

We can see in figure (8) that the system is stable, as the errors are decreasing through the platoon. We can see also that the maximum error is smaller than \( L \).

When comparing our control law and the classical CTH law we can see in figure (7) and figure (8) that the distances
between vehicles have been reduced from the range [5-40] meters for CTH to the range [0.5-1.5] meters using our control law. In addition, we can see that the system becomes faster.

**V. DISCUSSION**

The proposed approach greatly reduces inter-vehicle distances required, while assuring stability. This is obtained by making the distance proportional, not to velocity, but to the difference between the vehicle velocity and a common velocity value shared by all vehicles of the platoon.

**A. Advantages and comparison**

Using the new spacing policy and the corresponding new control law, the advantages are the following:

**String stability**: The propagation function $G(s)$, corresponding to the new control law, is not related to $V$, so the value of $V$ will not affect the platoon stability. It can be noticed that it is exactly the same propagation transfer function as the classical time headway spacing policy, so with this modification the system remains string stable.

**Inter-vehicle distances**: The most important effect of the proposed modification is on the inter-vehicle distances. At equilibrium, if all velocities become equal to leader velocity $v_L$ then $\Delta X_i = L + h (v_L - V)$. By choosing $V = v_L$ the inter-vehicle distance becomes $\Delta X_i = L$, and during dynamic changes the inter-vehicle distance becomes $\Delta X_i = L + h (v_i - V)$.

The inter-vehicle distance has been decreased from $\Delta X_i = L + h v_i$ (which might be very large at high speeds) in the case of the classical time headway policy [14], [6], to become $\Delta X_i = L + h (v_i - V)$, which is equal to $L$ at equilibrium and slightly larger than $L$ during transient phases. So during transient phases, the length of the platoon will be slightly different from the length of a platoon using constant spacing policy [17], [13].

Another important point is the effect of increasing parameter $h$, which has a positive effect on stability. In CTH it has a large negative effect on the inter-vehicle distance, as this distance increases proportionally to $h$, and hence the traffic density decreases. In our case, the inter distance is also proportional to $h$ but with a smaller coefficient $(v_i - V)$, so the inter distance changes will be smaller than the changes in CTH.

**Collisions**: it is clear that the possibility of a collision between the vehicles is increased as the inter distance between them is reduced. The problem of collision can be addressed separately from the problem of stability by adding a new term for the safety, but we have proved in this article that the platoon is string stable and safe in normal working conditions with small inter-vehicle distances.

**Communication**: Adding $V$ to the control law impose exchanging data between the vehicles. We have seen previously that stability is not related to $V$, so the rate of exchanged data between the vehicles can be reduced by updating the value of $V$ every sample times according to the change rate of $V$ as it will be discussed later.

**Stability without communication**: The string stability can be preserved even if the communication with the leader is totally lost, by switching to the classical time headway policy, which corresponds to setting $V = 0$ (fully autonomous mode). In this case, there is no need to communicate with the leader. So this law can keep the platoon stable even if communication is lost. On the contrary, it has been proved that the constant spacing policy can not be string stable, for homogeneous platoon with homogeneous control (all the gains are equals), without using any information from other vehicles [10].

Hand shaking protocol, between the leader and other vehicles, is very important to detect any loss of communication. If any loss is detected, the leader will transmit an order to all vehicles to switch to full autonomous mode $V = 0$, while the vehicle which has lost communication, will automatically switch to this mode when it detects the communication loss.
Simplicity and type of required data: The new control has the same simplicity as CTH law. It uses the same variables as CTH, plus a low frequency updating of the common speed parameter \( V \) (which may be the leader or platoon speed). This last variable is the only difference with the classical time headway policy, while the constant spacing policy is always more complicated, as it may require the acceleration or other information, at least from the leader.

B. Supervision of parameter \( V \)

As seen previously, the only condition to keep the platoon stable with the new control law is to make \( V \) identical for all the vehicles at any sample time. So, any value for \( V \) (e.g. leader’s velocity, the medium velocity of the platoon or the minimum velocity in the platoon...) can be chosen.

To increase the safety and to prevent collisions, one can choose \( V = \min(v_{\text{Leader}}, v_1, v_2, \ldots, v_N) \). This will always make \( h_v (v_i - V) > 0 \). In that case, the inter vehicle distance becomes \( \Delta X = L + h_v (v_i - V) > L \) but of course it will enlarge the inter-vehicle distance during velocity changes.

The rate of updating \( V \) define the rate of exchanged data between vehicles. Reducing this rate will improve our control law by avoiding the need for high rate communication and reducing the effects of transmission delays and data lose. The rate of changes of \( V \) is usually lower than the rate of the changes of \( v_i, a_i, i = 1 \ldots N \), so the update rate of \( V \) can be lower than the sampling rate of control laws of each vehicle. But, lowering the update rate may produce some jumps in \( V \), which may have negative effects on the control and hence on the performance. So \( V \) must be interpolated to get smoother changes.

VI. CONCLUSIONS

In this paper, the design of longitudinal control of platoons in highways has been addressed. We have improved the response of our control law proposed in [1] by taking into account the model of the motor of the vehicle. This enabled to reduce the distance between the vehicles down to 1 meter without losing the stability and the safety of the platoon when working in the normal conditions. We have proved also that most of the properties still correct for the model of third degree for the vehicle. All the provided results have been tested under TORCS to check the validity of the proposed approach.

REFERENCES